Finney Demana Waits Kennedy Bressoud

# Calculus

AP® Edition

## Graphical, Numerical, Algebraic

Fifth Edition

# Calculus

## Graphical, Numerical, Algebraic

FIFTH EDITION

AP® Edition

# Calculus

## Graphical, Numerical, Algebraic

FIFTH EDITION

AP® Edition


Ross L. Finney

Franklin D. Demana
THE OHIO STATE UNIVERSITY

Bert K. Waits
THE OHIO STATE UNIVERSITY

Daniel Kennedy
BAYLOR SCHOOL

David M. Bressoud
MACALESTER COLLEGE



PEARSON

Boston Columbus Hoboken Indianapolis New York San Francisco Amsterdam
Cape Town Dubai London Madrid Milan Munich Paris Montreal Toronto Delhi
Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

| | |
|---|---|
| Editor in Chief | Anne Kelly |
| Manager, Product Management and Marketing | Jackie Flynn |
| Assistant Editor | Judith Garber |
| Program Manager | Danielle Miller |
| Project Manager | Mary Sanger |
| Program Management Team Lead | Marianne Stepanian |
| Project Management Team Lead | Christina Lepre |
| Media Producer | Jean Choe |
| MathXL Content Developer | Kristina Evans |
| Senior Author Support/ Technology Specialist | Joe Vetere |
| Rights and Permissions Project Manager | Maureen Griffin |
| Procurement Specialist | Carol Melville |
| Associate Director of Design | Andrea Nix |
| Program Design Lead | Beth Paquin |
| Text Design, Composition, Illustrations, Production Coordination | Cenveo® Publisher Services |
| Cover Design | Cenveo® Publisher Services |
| Cover Image | Jule Berlin/Shutterstock |



**Library of Congress Cataloging-in-Publication Data**

Finney, Ross L., author.

Calculus : graphical, numerical, algebraic / Ross L. Finney, Franklin D. Demana, The Ohio State University, Bert K. Waits, The Ohio State University, Daniel Kennedy, Baylor School, David M. Bressoud. – 5th edition.

pages cm

ISBN 0-13-331161-9 – ISBN 0-13-331162-7

1. Calculus–Textbooks. I. Demana, Franklin D., 1938- author. II. Waits, Bert K., author. III. Kennedy, Daniel, 1946- author. IV. Title.

QA303.2.F547 2016

515–dc23 2014031803

1 2 3 4 5 6 7 8 9 10—CRK— 19 18 17 16 15

PEARSON

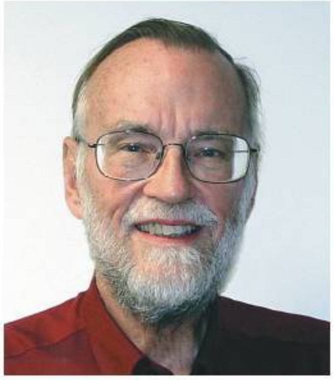
www.pearsonschool.com/advanced

ISBN 13: 978-0-13-331161-7 (high school binding)

ISBN 10: 0-13-331161-9 (high school binding)

# Dedication

This fifth edition of our calculus book is dedicated to the memory of our friend and coauthor, Bert K. (Hank) Waits, whose larger-than-life personality and infectious enthusiasm for mathematics education inspired two generations of teachers and, through them, millions of mathematics students.



# Foreword

If you could wave a magic wand and assemble your dream team of authors for an AP Calculus textbook, you would be very pleased if the group included the likes of Ross Finney, Frank Demana, Bert Waits, Dan Kennedy, and David Bressoud. I feel blessed to have known and worked closely with every one of them from as far back as 1989. With a pedigree that includes George Thomas in the storied authorship bloodline, this book represents the finest combination of excellent writing, creative insight, terrific problems, and captivating explorations. A dream team indeed!

Any teacher in search of a textbook for use in an AP Calculus class should look critically at whether or not the book is faithful to the philosophy and goals of the course, as well as the topic outline, each described in the AP Calculus Course Description. On both accounts, the FDWKB text shines brightly. True to its title, care is taken to attend to a genuine multirepresentational approach. You'll find problems and discourse involving graphs, tables, symbolic definitions, and verbal descriptions. And the text covers every topic that is tested on the AP exams. Dan Kennedy, who joined the author team with the 2003 publication, and David Bressoud, its most recent addition, both contribute an intimate knowledge of the AP Calculus course. The two share a familiarity with the program borne from years of service. Back in the mid 1990s, when I saw Dan describe how to teach series with an approach that ultimately became an integral part of Chapter 10 of the current edition, I knew it was the way to go. My students have used this text, and that approach, for many years with great success. David's presence on the team brings his passion both for the history of calculus and for its careful and precise presentation. He describes the contributions to the development of calculus by great thinkers that preceded Newton and Leibniz. He also illuminates the idea of using the derivative as a measure of sensitivity.

This text is a proven winner that just keeps getting better. With the new course framework rolling out, it is a perfect fit for any AP Calculus class today, and promises to be for years to come.

Mark Howell
Gonzaga College High School
Washington, DC

*Mark Howell has taught AP Calculus at Gonzaga High School for more than thirty years. He has served the AP Calculus community at the AP Reading as a reader, table leader, and question leader for eighteen years, and for four years he served as a member of the AP Calculus Development Committee. A College Board consultant for more than twenty years, Mark has led workshops and summer institutes throughout the United States and around the world. In 1993 he won a state Presidential Award in the District of Columbia, and in 1999 he won a Tandy Technology Scholars Award and the Siemens Foundation Award for Advanced Placement Teachers. He is the author of the current* AP Teacher's Guide for AP Calculus, *and is co-author with Martha Montgomery of the popular AP Calculus review book* Be Prepared for the AP Calculus Exam *from Skylight Publishing.*

# About the Authors

## Ross L. Finney

Ross Finney received his undergraduate degree and Ph.D. from the University of Michigan at Ann Arbor. He taught at the University of Illinois at Urbana–Champaign from 1966 to 1980 and at the Massachusetts Institute of Technology (MIT) from 1980 to 1990. Dr. Finney worked as a consultant for the Educational Development Center in Newton, Massachusetts. He directed the Undergraduate Mathematics and its Applications Project (U MAP) from 1977 to 1984 and was founding editor of the *UMAP Journal*. In 1984, he traveled with a Mathematical Association of America (MAA) delegation to China on a teacher education project through People to People International.

Dr. Finney co-authored a number of Addison-Wesley textbooks, including *Calculus; Calculus and Analytic Geometry; Elementary Differential Equations with Linear Algebra;* and *Calculus for Engineers and Scientists*. Dr. Finney's co-authors were deeply saddened by the death of their colleague and friend on August 4, 2000.

## Franklin D. Demana

Frank Demana received his master's degree in mathematics and his Ph.D. from Michigan State University. Currently, he is Professor Emeritus of Mathematics at The Ohio State University. As an active supporter of the use of technology to teach and learn mathematics, he is co-founder of the international Teachers Teaching with Technology ($T^3$) professional development program. He has been the director and co-director of more than $10 million of National Science Foundation (NSF) and foundational grant activities, including a $3 million grant from the U.S. Department of Education Mathematics and Science Educational Research program awarded to The Ohio State University. Along with frequent presentations at professional meetings, he has published a variety of articles in the areas of computer- and calculator-enhanced mathematics instruction. Dr. Demana is also co-founder (with Bert Waits) of the annual International Conference on Technology in Collegiate Mathematics (ICTCM). He is co-recipient of the 1997 Glenn Gilbert National Leadership Award presented by the National Council of Supervisors of Mathematics, and co-recipient of the 1998 Christofferson-Fawcett Mathematics Education Award presented by the Ohio Council of Teachers of Mathematics.

Dr. Demana co-authored *Precalculus: Graphical, Numerical, Algebraic; Essential Algebra: A Calculator Approach; Transition to College Mathematics; College Algebra and Trigonometry: A Graphing Approach; College Algebra: A Graphing Approach; Precalculus: Functions and Graphs*; and *Intermediate Algebra: A Graphing Approach*.

## Bert K. Waits

Bert Waits received his Ph.D. from The Ohio State University and taught Ohio State students for many years before retiring as Professor Emeritus of Mathematics. Dr. Waits co-founded the international Teachers Teaching with Technology ($T^3$) professional development program and was co-director or principal investigator on several large projects funded by the National Science Foundation. Active in both the Mathematical Association of America and the National Council of Teachers of Mathematics, he published more than 70 articles in professional journals and conducted countless lectures, workshops, and minicourses on how to use computer technology to enhance the teaching and learning of mathematics. Dr. Waits was co-recipient of the 1997 Glenn Gilbert National Leadership Award presented by the National Council of Supervisors of Mathematics and of the 1998 Christofferson-Fawcett Mathematics Education Award presented by the Ohio Council of Teachers of Mathematics. He was the co-founder (with Frank Demana) of the ICTCM and was one of six authors of the high school portion of the groundbreaking 1989 *NCTM Standards.* Dr. Waits was hard at work on revisions for the fifth edition of this calculus textbook when he died prematurely on July 27, 2014, leaving behind a powerful legacy in the legions of teachers whom he inspired.

Dr. Waits coauthored *Precalculus: Graphical, Numerical, Algebraic; College Algebra and Trigonometry: A Graphing Approach; College Algebra: A Graphing Approach; Precalculus: Functions and Graphs*; and *Intermediate Algebra: A Graphing Approach*.

## Daniel Kennedy

Dan Kennedy received his undergraduate degree from the College of the Holy Cross and his master's degree and Ph.D. in mathematics from the University of North Carolina at Chapel Hill. Since 1973 he has taught mathematics at the Baylor School in Chattanooga, Tennessee, where he holds the Cartter Lupton Distinguished Professorship. Dr. Kennedy joined the Advanced Placement® Calculus Test Development Committee in 1986, then in 1990 became the first high school teacher in 35 years to chair that committee. It was during his tenure as chair that the program moved to require graphing calculators and laid the early groundwork for the 1998 reform of the Advanced Placement Calculus curriculum. The author of the 1997 *Teacher's Guide—AP® Calculus*, Dr. Kennedy has conducted more than 50 workshops and institutes for high school calculus teachers. His articles on mathematics teaching have appeared in the *Mathematics Teacher* and the *American Mathematical Monthly*, and he is a frequent speaker on education reform at professional and civic meetings. Dr. Kennedy was named a Tandy Technology Scholar in 1992 and a Presidential Award winner in 1995.

Dr. Kennedy coauthored *Precalculus: Graphical, Numerical, Algebraic; Prentice Hall Algebra I; Prentice Hall Geometry;* and *Prentice Hall Algebra 2.*

## David M. Bressoud

David Bressoud received his undergraduate degree from Swarthmore College and Ph.D. from Temple University. He taught at Penn State from 1977 to 1994, is currently DeWitt Wallace Professor of Mathematics at Macalester College, and is a former president of the Mathematical Association of America. He is the author of several textbooks on number theory, combinatorics, vector calculus, and real analysis, all with a strong historical emphasis. He taught AP Calculus at the State College Area High School in 1990–91, began as an AP Reader in 1993, and served on the AP Calculus Test Development Committee for six years and as its chair for three of those years. He currently serves on the College Board Mathematical Sciences Academic Advisory Committee. He has been Principal Investigator for numerous grants, including two large National Science Foundation grants to study Characteristics of Successful Programs in College Calculus and Progress Through Calculus. He also writes *Launchings,* a monthly blog on issues of undergraduate mathematics education.

# Contents

CHAPTER 1



CHAPTER 2

CHAPTER 10

## Infinite Series 480



CHAPTER 11

## Parametric, Vector, and Polar Functions 542

## APPENDICES

# To the Teacher

The fifth edition of *Calculus: Graphical, Numerical, Algebraic,* AP* Edition, by Finney, Demana, Waits, Kennedy, and Bressoud completely supports the content, philosophy, and goals of the Advanced Placement (AP*) Calculus courses (AB and BC).

The College Board has recently finished a lengthy and thorough review of the AP* Calculus courses to ensure that they continue to keep pace with the best college and university courses that are taught with similar educational goals. This review has resulted in a repackaging of the course descriptions in terms of big ideas, enduring understandings, learning objectives, and essential knowledge, but the learning goals remain essentially the same. That has allowed us to retain the overall flow of our previous edition and concentrate our attention on how we might be more helpful to you and your students in certain parts of the course.

A very broad look at the overall goals of this textbook is given in the following bulleted summary. Although these are not explicit goals of the AP program and do not include all of the learning objectives in the new AP Curriculum Framework, they do reflect the intentions of the AP Calculus program. (Note that the asterisked goals are aligned with the BC course and are not required in AB Calculus.)

- Students will be able to work with functions represented graphically, numerically, analytically, or verbally, and will understand the connections among these representations; graphing calculators will be used as a tool to facilitate such understanding.
- Students will, in the process of solving problems, be able to use graphing calculators to graph functions, solve equations, evaluate numerical derivatives, and evaluate numerical integrals.
- Students will understand the meaning of the derivative as a limit of a difference quotient and will understand its connection to local linearity and instantaneous rates of change.
- Students will understand the meaning of the definite integral as a limit of Riemann sums and as a net accumulation of change over an interval, and they will understand and appreciate the connection between derivatives and integrals.
- Students will be able to model real-world behavior and solve a variety of problems using functions, derivatives, and integrals; they will also be able to communicate solutions effectively, using proper mathematical language and syntax.
- Students will be able to represent and interpret differential equations geometrically with slope fields and (*) numerically with Euler's method; they will be able to model dynamic situations with differential equations and solve initial value problems analytically.
- (*) Students will understand the convergence and divergence of infinite series and will be able to represent functions with Maclaurin and Taylor series; they will be able to approximate or bound truncation errors in various ways.
- (*) Students will be able to extend some calculus results to the context of motion in the plane (through vectors) and to the analysis of polar curves.

The incorporation of graphing calculator technology throughout the course continues to be a defining feature of this textbook, but we urge teachers to read the next section, Philosophy on Technology Usage, to see how our philosophy has changed over time (again, in harmony with our AP* Calculus colleagues). Whether you are concerned about how to use calculators enough or how not to use calculators too much, we believe you can trust this author team to address your concerns with the perspective that only long experience can provide.

Whether you are a veteran user of our textbooks or are coming on board for this fifth edition, we thank you for letting us join you in the important adventure of educating your students. Some of the best suggestions for improving our book over the years have come from students and teachers, so we urge you to contact us through Pearson if you have any questions or concerns. To paraphrase Isaac Newton, if this textbook enables your students to see further down the road of mathematics, it is because we have stood on the shoulders of dedicated teachers like you.

# Philosophy on Technology Usage

When the AP Calculus committee, after consultation with leaders in mathematics and mathematics education, made the decision to require graphing calculators on the AP examinations, their intent was to enhance the teaching and learning of calculus in AP classrooms. In fact, that decision probably had an impact on the entire secondary mathematics curriculum, as teachers discovered that graphing utilities could make a deeper understanding of function behavior possible in earlier courses. The authors of this textbook have supported the use of this technology from its inception and, well aware of its potential for either facilitating or circumventing true understanding, have employed it carefully in our approach to problem solving. Indeed, longtime users of this textbook are well acquainted with our insistence on the distinction between **solving** the problem and **supporting** or **confirming** the solution, and how technology figures into each of those processes.

As hand-held graphing technology enters its fourth decade, we have come to realize that advances in technology and increased familiarity with calculators have gradually blurred some of the distinctions between solving and supporting that we had once assumed to be apparent. Textbook exercises that we had designed for a particular pedagogical purpose are now being solved with technology in ways that either side-step or obscure the learning we had hoped might take place. For example, students might find an equation of the line through two points by using linear regression, or they might match a set of differential equations with their slope fields by simply graphing each slope field. Now that calculators with computer algebra systems have arrived on the scene, exercises meant for practicing algebraic manipulations are being solved without the benefit of the practice. We do not want to retreat in any way from our support of modern technology, but we feel that the time has come to provide more guidance about the intent of the various exercises in our textbook.

Therefore, as a service to teachers and students alike, exercises in this textbook that **should be solved without calculators** will be identified with gray ovals around the exercise numbers. These will usually be exercises that demonstrate how various functions behave algebraically or how algebraic representations reflect graphical behavior and vice versa. Application problems will usually have no restrictions, in keeping with our emphasis on **modeling** and on bringing **all representations** to bear when confronting real-world problems. We are pleased to note that this all meshes well with the Mathematical Practices for AP Calculus (MPACs).

When the AP Calculus committee decided to change the number of noncalculator problems on the free-response portions of the AP examinations from three to four (out of six), their concern was related to test development rather than pedagogy. Still, their decision, like ours, was informed by years of experience with the teachers, the students, and the technology. The goal today is the same as it was thirty years ago: to enhance the teaching and learning of mathematics.

Incidentally, we continue to encourage the use of calculators to **support** answers graphically or numerically after the problems have been solved with pencil and paper. Any time students can make those connections among the graphical, analytical, and numerical representations, they are doing good mathematics. We just don't want them to miss something along the way because they brought in their calculators too soon.

As a final note, we will freely admit that different teachers use our textbook in different ways, and some will probably override our no-calculator recommendations to fit with their pedagogical strategies. In the end, the teachers know what is best for their students, and we are just here to help.

# Changes for This Edition

The big change in this new edition, of course, is that we welcome David M. Bressoud to the author team. As a college mathematician with years of experience with the Advanced Placement (AP) program, he is well acquainted with the scope and the goals of the course, and he is well versed in the mathematics that calculus students should know. As an experienced writer with a particular passion for the history of mathematics, he can provide contexts that make calculus topics more interesting for students and teachers alike. Teachers who have used our previous editions will have no trouble identifying (and, we hope, enjoying) the ways that David has enriched what was already a gratifyingly popular book.

## A Few Global Changes

As with previous editions, we have kept our focus on high school students taking one of the College Board's AP courses, Calculus AB or Calculus BC. The topics in the book reflect both the curriculum and the pedagogy of an AP course, and we do what we can to prepare students for the AP examinations throughout the book. As the College Board has moved toward objective-based course descriptions, we have replaced our "What You Will Learn . . . and Why" feature in each section opener by learning objectives and bulleted context topics. Also, as free-response questions on the AP examinations have evolved in focus and style, we have added or amended our exercises to reflect those changes. Although the role of graphing calculators in the course has not changed significantly since the fourth edition, we have made a few changes based on calculator evolution. We have also removed some of the gray "no calculator" ovals from the fourth edition to reflect an emerging consensus among calculus teachers that *not every* problem that can be solved without a calculator *ought to* be solved without a calculator.

We have expanded the treatment of the derivative as a measure of *sensitivity*, which now appears as an ongoing topic in several different sections of the book. We have also made a more consistent commitment to *point-slope form* for linear equations throughout the book, as that is the form that emphasizes the concept of local linearity, critical for understanding differential calculus.

We decided in this edition to eliminate all references to calculator regression functionality and all solutions that involved fitting curves to data points. There are still plenty of examples and exercises based on numerical data and tables, but the solutions do not involve first finding functions to fit the points to allow for an analytic solution. Calculator regression has never been permissible on AP calculus examinations, precisely because it changes the intent of the problem. We therefore felt that our regression references might be doing more harm than good.

Most of the motivational "Chapter Opener" problems are new and improved in this edition, and (thanks to David) many more historical nuggets have been sprinkled throughout the book. We also confess that some of the nuggets that were already there were in need of refinement. For example, the name of one of the key players has been restored to its original spelling: Guillaume de l'Hospital.

Finally, be assured that we have looked carefully at the content of the new AP Calculus Curriculum Framework, and there is nothing therein that is not covered in this book. Moreover, the emphases are in harmony with the objectives of the AP course, as they have always been.

## Some Specific Chapter Changes

Chapter 1, which is really a review of precalculus topics, has been altered in several places to be less of a "here is a summary of what you have learned" experience and more of a "here are some things you have learned that you will need for studying calculus" experience. For example, the section on lines is now about linear *functions* and makes references to their importance in calculus—despite the fact that it appears before the section that reviews functions. We have also added a subsection on solving simultaneous linear equations (including by matrices with a calculator).

The Chapter 2 opener has been changed to introduce the topic of sensitivity, the measurement of how change in the input variable affects change in the output variable, and a subsection with examples about sensitivity has been added. The epsilon-delta definition of limit (not an AP topic, and no longer assumed in many first-year college courses) has been entirely moved to the appendices, while the intuitive definition has been strengthened and clarified. In line with the AP course descriptions, the Sandwich Theorem has been renamed the Squeeze Theorem.

In most cases, linear equations appearing as answers in examples and exercises in Chapter 3 have been converted to point-slope form to reinforce the idea of local linearity. Additional attention is now paid to the "MathPrint" version of the numerical derivative, while retaining references to what is now called the "classic" syntax.

A new example has been added to the end of Chapter 4 to illustrate (in multiple-choice form) the most common errors students make when applying, or failing to apply, the Chain Rule.

In Chapter 5, examples and exercises have been added to illustrate sensitivity as an application of the derivative. A proof of the Mean Value Theorem and an exploration of Lagrange's bound on the difference between a function and its tangent line approximation have been added as exercises in *Extending the Ideas*.

Chapter 6 now contains more coverage of *accumulation* as a fundamental interpretation of integration, and more attention is paid to the *accumulator function* $\int_a^x f(t)\,dt$ and its applications. The two parts of the Fundamental Theorem of Calculus have been helpfully distinguished as the Antiderivative Part and the Evaluation Part.

In Chapter 7, the example showing how to handle discontinuity in an initial value problem has been modified to show more clearly why the domain of the solution must be restricted. Exercises involving exponential and logistic regression have been replaced with additional exercises involving separation of variables and the solution to the general logistic equation. More free-response questions involving slope fields and separation of variables have been added to the chapter review.

More exercises and examples on accumulation have been added to Chapter 8. Another example and some exercises have been added to give students more familiarity with solids of revolution around lines that are not the coordinate axes.

In Chapter 9, more material has been added on the Fibonacci sequence and the golden ratio $\varphi$. Graphing calculator screens have been updated to reflect the newer sequence format and to show how the TRACE feature can be used to explore limits of sequences.

In Chapter 10, a margin note has been added to make term-by-term differentiation of power series simpler for students. Section 3 (Taylor's Theorem) has been extensively revised to make truncation error analysis easier for BC students. The alternating series error bound is now introduced in this section so it can be compared with the Lagrange error bound in examples and exercises, and more AP-style problems involving truncation error analysis have been added. The Remainder Estimation Theorem has been renamed the Remainder Bounding Theorem to reflect how the theorem is actually used.

Chapter 11 now includes a discussion of how polar equations can be used to describe conic sections. There is also a brief introduction to the use of conic sections in orbital mechanics, along with several new exercises in that context. Some interesting new historical references have been added to explain how certain polar curves acquired their names.

Finally, we have performed some appendectomies on the material in the back of the book that has been accruing over the years. We removed the appendices on Mathematical Induction and Conic Sections, as those are really precalculus topics. We also removed nearly half of the formulas in the Brief Table of Integrals, retaining only those that were within the scope of the course, or interesting, or both.

# Continuing Features

## Mathematical Practices for AP Calculus (MPACs)

The Mathematical Practices highlighted in the *AP Calculus Curriculum Framework* always have been and continue to be represented in the text, examples, and exercises.

**MPAC 1, Reasoning with definitions and theorems** is one of the dominant themes in the development of each new idea and of the exercises. Definitions and theorems are highlighted in each section and summarized at the end of each chapter for reference and review.

**MPAC 2, Connecting concepts** runs throughout this book, introducing new concepts by connecting them to what has come before and in the reliance of many exercises that draw on applications or build on student knowledge. *Quick Review* exercises at the start of each Exercise set review concepts from previous sections (or previous courses) that will be needed for the solutions.

**MPAC 3, Implementing algebraic/computational processes** is well represented in the foundational exercises with which each exercise set begins and in the thoughtful use of technology.

**MPAC 4, Connecting multiple representations** has always been present in the emphasis on the connections among graphical, numerical, and algebraic representations of the key concepts of calculus. The title of this book speaks for itself in that regard.

**MPAC 5, Building notational fluency** is represented in the intentional use of a variety of notational forms and in their explicit connection to graphical, numerical, and algebraic representations. Many margin notes explicitly address notational concerns.

**MPAC 6, Communicating** is a critical component of the Explorations that appear in each section. Communication is also essential to the *Writing to Learn* exercises as well as the *Group Activities.* Many of the exercises and examples in the book have "justify your answer" components in the spirit of the AP exams.

## Balanced Approach

A principal feature of this edition is the balance attained among the rule of four: analytic/algebraic, numerical, graphical, and verbal methods of representing problems. We believe that students must value all of these methods of representation, understand how they are connected in a given problem, and learn how to choose the one(s) most appropriate for solving a particular problem. **(MPACs 2 and 4)**

## The Rule of Four

In support of the rule of four, we use a variety of techniques to solve problems. For instance, we obtain solutions algebraically or analytically, support our results graphically or numerically with technology, and then interpret the result in the original problem context. We have written exercises in which students are asked to solve problems by one method and then support or confirm their solutions by using another method. We want students to understand that technology can be used to support (but not prove) results, and that algebraic or analytic techniques are needed to prove results. We want students to understand that mathematics provides the foundation that allows us to use technology to solve problems. **(MPACs 1, 3, 4, and 5)**

## Applications

The text includes a rich array of interesting applications from biology, business, chemistry, economics, engineering, finance, physics, the social sciences, and statistics. Some applications are based on real data from cited sources. Students are exposed to functions as mechanisms for modeling data and learn about how various functions can model real-life problems. They learn to analyze and model data, represent data graphically, interpret from graphs, and fit curves. Additionally, the tabular representations of data presented in the text highlight the concept that a function is a correspondence between numerical variables, helping students to build the connection between the numbers and the graphs. **(MPACs 2, 4, and 6)**

## Explorations

Students are expected to be actively involved in understanding calculus concepts and solving problems. Often the explorations provide a guided investigation of a concept. The explorations help build problem-solving ability by guiding students to develop a mathematical model of a problem, solve the mathematical model, support or confirm the solution, and interpret the solution. The ability to communicate their understanding is just as important to the learning process as reading or studying, not only in mathematics but also in every academic pursuit. Students can gain an entirely new perspective on their knowledge when they explain what they know, either orally or in writing. **(MPACs 1, 2, 4, and 6)**

## Graphing Utilities

The book assumes familiarity with a graphing utility that will produce the graph of a function within an arbitrary viewing window, find the zeros of a function, compute the derivative of a function numerically, and compute definite integrals numerically. Students are expected to recognize that a given graph is reasonable, identify all the important characteristics of a graph, interpret those characteristics, and confirm them using analytic methods. Toward that end, most graphs appearing in this book resemble students' actual grapher output or suggest hand-drawn sketches. This is one of the first calculus textbooks to take full advantage of graphing calculators, philosophically restructuring the course to teach new things in new ways to achieve new understanding, while (courageously) abandoning some old things and old ways that are no longer serving a purpose. **(MPACs 3. 4 and 5)**

## Exercise Sets

The exercise sets were updated for this edition, including many new ones. There are nearly 4000 exercises, with more than 80 Quick Quiz exercises and 560 Quick Review exercises. The different types of exercises included are Algebraic and analytic manipulation, Interpretation of graphs, Graphical representations, Numerical representations, Explorations, Writing to learn, Group activities, Data analyses, Descriptively titled applications, Extending the ideas.

Each exercise set begins with the Quick Review feature, which can be used to introduce lessons, support Examples, and review prerequisite skills. The exercises that follow are graded from routine to challenging. Some exercises are also designed to be solved *without a calculator*; these exercises have numbers printed within a gray oval. Students are urged to **support** the answers to these (and all) exercises graphically or numerically, but only after they have solved them with pencil and paper. An additional block of exercises, Extending the Ideas, may be used in a variety of ways, including group work. We also provide Review Exercises and AP Examination Preparation at the end of each chapter.

# Print Supplements and Resources

## For the Student

*The following supplements are available for purchase:*

**AP* Test Prep Series: AP* Calculus**
**(ISBN: 0133314588)**

- Introduction to the AP AB and BC Calculus Exams
- Precalculus Review of Calculus Prerequisites
- Review of AP Calculus AB and Calculus BC Topics
- Practice Exams
- Answers and Solutions

## For the Teacher

*The following supplements are available to qualified adopters:*

**Annotated Teacher's Edition**

- Answers included on the same page as the problem appears, for most exercises. All answers included in the back of the book.
- Solutions to Chapter Opening Problems, Teaching Notes, Common Errors, Notes on Examples and Exploration Extensions, and Assignment Guide included at the beginning of the book.

**Solutions Manual**

- Complete solutions for Quick Reviews, Exercises, Explorations, and Chapter Reviews

# Technology Resources

The Fifth Edition of Finney, Demana, Waits, Kennedy, Bressoud *Calculus* is accompanied by an extensive range of technology resources designed to support students in practicing and learning the material, and to assist teachers in managing and delivering their courses.

***MathXL® for School (optional, for purchase only)—access code required, www.mathxlforschool.com***

MathXL for School is a powerful online homework, tutorial, and assessment supplement that aligns to Pearson Education's textbooks in mathematics or statistics. With MathXL for School, teachers can:

- Create, edit, and assign auto-graded online homework and tests correlated at the objective level to the textbook
- Utilize automatic grading to rapidly assess student understanding
- Track both student and group performance in an online gradebook
- Prepare students for high-stakes testing, including aligning assignments to state and Common Core State Standards, where available
- Deliver quality, effective instruction regardless of experience level

With MathXL for School, students can:

- Do their homework and receive immediate feedback
- Get self-paced assistance on problems in a variety of ways (guided solutions, step-by-step examples, video clips, animations)
- Have a large number of practice problems to choose from, helping them master a topic
- Receive personalized study plans and homework based on test results

For more information and to purchase student access codes after the first year, visit our Web site at www.mathxlforschool.com, or contact your Pearson Account General Manager.

***MyMathLab® Online Course (optional, for purchase only)—access code required, www.mymathlab.com***

MyMathLab is a text-specific, easily customizable, online course that integrates interactive multimedia instruction with textbook content. MyMathLab gives you the tools you need to deliver all or a portion of your course online.

MyMathLab features include:

- Interactive eText, including highlighting and note taking tools, and links to videos and exercises
- Rich and flexible course management, communication, and teacher support tools
- Online homework and assessment, and personalized study plans
- Complete multimedia library to enhance learning
- All teacher resources in one convenient location

For more information, visit www.mymathlabforschool.com or contact your Pearson Account General Manager.

#### *Video Resources*

These video lessons feature an engaging team of mathematics teachers who present comprehensive coverage of each section of the text. The lecturers' presentations include examples and exercises from the text and support an approach that emphasizes visualization and problem solving. Available in MyMathLab for School.

### Additional Support for Teachers

Most of the teacher supplements and resources for this book are available electronically upon adoption or to preview. For more information, please contact your Pearson School sales representative.

#### *Downloadable Teacher's Resources*

- **Texas Instruments Graphing Calculator Manual** is an introduction to Texas Instruments' graphing calculators, as they are used for calculus. Featured are the TI-84 Plus Silver Edition with MathPrint, the TI-83 Plus Silver Edition, and the TI-89 Titanium. The keystrokes, menus, and screens for the TI-84 Plus are similar to the TI-84 Plus Silver Edition; those for the TI-83 Plus are similar to the TI-83 Plus Silver Edition; and the TI-89, TI-92 Plus, and Voyage™ 200 are similar to the TI-89 Titanium.
- **Resources for AP* Exam Preparation and Practice** includes many resources to help prepare students for the AP* Calculus exam, including concepts worksheets, sample AB and BC exams, and answers to those exams.
- **AP* Calculus Implementation Guide** is a tool to help teachers manage class time and ensure the complete Advanced Placement* Calculus Curriculum Framework is covered. It includes pacing guides (for AB and BC Calculus), assignment guides, topic correlations, and lesson plans.
- **PowerPoint® Lecture Presentation** This time-saving resource includes classroom presentation slides that align to the topic sequence of the textbook.
- **Assessment Resources** include chapter quizzes, chapter tests, semester test, final tests, and alternate assessments.

#### *TestGen®*

TestGen enables teachers to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing teachers to create multiple but equivalent versions of the same question or test with the click of a button. Teachers can also modify test bank questions or add new questions. Tests can be printed or administered online.

# To the AP Student

We know that as you study for your AP course, you're preparing along the way for the AP exam. By tying the material in this book directly to AP course goals and exam topics, we help you to focus your time most efficiently. And that's a good thing!

The AP exam is an important milestone in your education. A high score will position you optimally for college acceptance—and possibly will give you college credits that put you a step ahead. Our primary commitment is to provide you with the tools you need to excel on the exam . . . the rest is up to you!

## Test-Taking Strategies for an Advanced Placement Calculus Examination

You should approach the AP Calculus Examination the same way you would any major test in your academic career. Just remember that it is a one-shot deal—you should be at your peak performance level on the day of the test. For that reason you should do everything that your "coach" tells you to do. In most cases your coach is your classroom teacher. It is very likely that your teacher has some experience, based on workshop information or previous students' performance, to share with you.

You should also analyze your own test-taking abilities. At this stage in your education, you probably know your strengths and weaknesses in test-taking situations. You may be very good at multiple choice questions but weaker in essays, or perhaps it is the other way around. Whatever your particular abilities are, evaluate them and respond accordingly. Spend more time on your weaker points. In other words, rather than spending time in your comfort zone where you need less work, try to improve your soft spots. In all cases, concentrate on clear communication of your strategies, techniques, and conclusions.

The following table presents some ideas in a quick and easy form.

**General Strategies for AP Examination Preparation**

| Time | Dos |
|---|---|
| Through the Year | • Register with your teacher/coordinator<br>• Pay your fee (if applicable) on time<br>• Take good notes<br>• Work with others in study groups<br>• Review on a regular basis<br>• Evaluate your test-taking strengths and weaknesses |
| Several Weeks Before | • Combine independent and group review<br>• Get tips from your teacher<br>• Do lots of mixed review problems<br>• Check your exam date, time, and location<br>• Review the appropriate AP Calculus syllabus (AB or BC)<br>• Make sure your calculator is on the approved list |
| The Night Before | • Put new batteries in your calculator or make sure it is charged<br>• Set your calculator in Radian Mode<br>• Lay out your clothes and supplies so that you are ready to go out the door<br>• Do a short review<br>• Go to bed at a reasonable hour |
| Exam Day | • Get up a little earlier than usual<br>• Eat a good breakfast/lunch<br>• Get to your exam location 15 minutes early |
| Exam Night | • Relax—you have earned it |

## Topics from the Advanced Placement Curriculum for Calculus AB, Calculus BC

As an AP student, you are probably well aware of the good study habits that are needed to be a successful student in high school and college:

- attend all the classes
- ask questions (either during class or after)
- take clear and understandable notes
- make sure you are understanding the concepts rather than memorizing formulas
- do your homework; extend your test-prep time over several days or weeks, instead of cramming
- use all the resources—text and people—that are available to you.

No doubt this list of "good study habits" is one that you have seen or heard before. You should know that there is powerful research that suggests a few habits or routines will enable you to go beyond "knowing about" calculus, to more deeply "understanding" calculus. Here are three concrete actions for you to consider:

- Review your notes at least once a week and rewrite them in summary form.
- Verbally explain concepts (theorems, etc.) to a classmate.
- Form a study group that meets regularly to do homework and discuss reading and lecture notes.

Most of these tips boil down to one mantra, which all mathematicians believe in:

*Mathematics is not a spectator sport.*

The AP Calculus Examination is based on the following Topic Outline. For your convenience, we have noted all Calculus AB and Calculus BC objectives with clear indications of topics required only by the Calculus BC Exam. The outline cross-references each AP Calculus objective with the appropriate section(s) of this textbook: *Calculus: Graphical, Numerical, Algebraic,* AP* Edition, Fifth Edition, by Finney, Demana, Waits, Kennedy, and Bressoud.

Use this outline to track your progress through the AP exam topics. Be sure to cover every topic associated with the exam you are taking. Check it off when you have studied and/or reviewed the topic.

Even as you prepare for your exam, I hope this book helps you map—and enjoy—your calculus journey!

—John Brunsting
*Hinsdale Central High School*

# Concept Outline for AP Calculus AB and AP Calculus BC

(excerpted from the College Board's Curriculum Framework—AP Calculus AB and AP Calculus BC, Fall 2014)

**EU = Enduring Understanding, LO = Learning Objective, BC only topics**

## Big Idea 1: Limits

| | *Sections* |
|---|---|
| **EU 1.1:** The concept of a limit can be used to understand the behavior of functions. | |
| **LO 1.1A(a):** Express limits symbolically using correct notation. | 2.1, 2.2 |
| **LO 1.1A(b):** Interpret limits expressed symbolically. | 2.1, 2.2 |
| **LO 1.1B:** Estimate limits of functions. | 2.1, 2.2 |
| **LO 1.1C:** Determine limits of functions. | 2.1, 2.2, 9.2, 9.3 |
| **LO 1.1D:** Deduce and interpret behavior of functions using limits. | 2.1, 2.2, 9.3 |
| **EU 1.2:** Continuity is a key property of functions that is defined using limits. | |
| **LO 1.2A:** Analyze functions for intervals of continuity or points of discontinuity. | 2.3 |
| **LO 1.2B:** Determine the applicability of important calculus theorems using continuity. | 2.3, 5.1, 5.2, 6.2–4 |

## Big Idea 2: Derivatives

| | *Sections* |
|---|---|
| **EU 2.1:** The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies. | |
| **LO 2.1A:** Identify the derivative of a function as the limit of a difference quotient. | 3.1 |
| **LO 2.1B:** Estimate the derivative. | 3.1, 3.2 |
| **LO 2.1C:** Calculate derivatives. | 3.3, 3.5, 4.1–4, 11.1–3 |
| **LO 2.1D:** Determine higher order derivatives. | 3.3, 4.2 |
| **EU 2.2:** A function's derivative, which is itself a function, can be used to understand the behavior of the function. | 2.4 |
| **LO 2.2A:** Use derivatives to analyze properties of a function. | 5.1–3, 11.1–3 |
| **LO 2.2B:** Recognize the connection between differentiability and continuity. | 3.2 |
| **EU 2.3:** The derivative has multiple interpretations and applications including those that involve instantaneous rates of change. | |
| **LO 2.3A:** Interpret the meaning of a derivative within a problem. | 2.4, 3.1, 3.4, 5.5 |
| **LO 2.3B:** Solve problems involving the slope of a tangent line. | 2.4, 3.4, 5.5 |
| **LO 2.3C:** Solve problems involving related rates, optimization, rectilinear motion, (BC) and planar motion. | 3.4, 5.1, 5.3, 5.4, 5.6, 11.1–3 |
| **LO 2.3D:** Solve problems involving rates of change in applied contexts. | 5.5, 5.6 |
| **LO 2.3E:** Verify solutions to differential equations. | 7.1 |
| **LO 2.3F:** Estimate solutions to differential equations. | 7.1 |
| **EU 2.4:** The Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval. | |
| **LO 2.4A:** Apply the Mean Value Theorem to describe the behavior of a function over an interval. | 5.2 |

## Big Idea 3: Integrals and the Fundamental Theorem of Calculus

| | *Sections* |
|---|---|
| **EU 3.1:** Antidifferentiation is the inverse process of differentiation. | |
| **LO 3.1A:** Recognize antiderivatives of basic functions. | 6.3 |

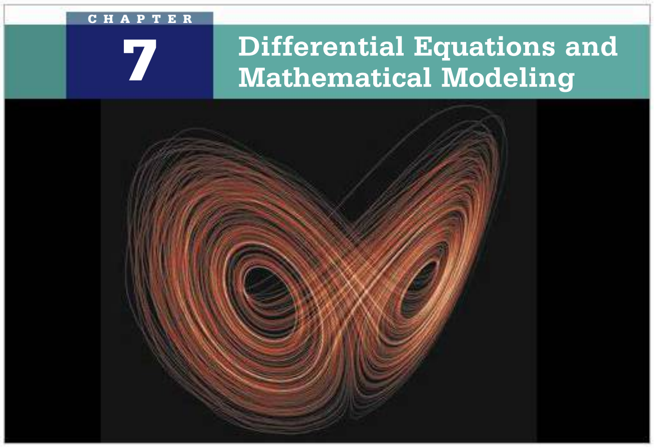
| Big Idea 3: Integrals and the Fundamental Theorem of Calculus—continued | *Sections* |
|---|---|
| **EU 3.2:** The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies. | |
| **LO 3.2A(a):** Interpret the definite integral as the limit of a Riemann sum. | 6.1, 6.2 |
| **LO 3.2A(b):** Express the limit of a Riemann sum in integral notation. | 6.2 |
| **LO 3.2B:** Approximate a definite integral. | 6.1, 6.2, 6.5 |
| **LO 3.2C:** Calculate a definite integral using areas and properties of definite integrals. | 6.2, 6.3 |
| **LO 3.2D: (BC)** Evaluate an improper integral or show that an improper integral diverges. | 9.4 |
| **EU 3.3:** The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration. | |
| **LO 3.3A:** Analyze functions defined by an integral. | 6.1–4, 8.1 |
| **LO 3.3B(a):** Calculate antiderivatives. | 6.3, 6.4, 7.2, 7.3, 7.5 |
| **LO 3.3B(b):** Evaluate definite integrals. | 6.3, 6.4, 7.2, 7.3, 7.5 |
| **EU 3.4:** The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation. | |
| **LO 3.4A:** Interpret the meaning of a definite integral within a problem. | 6.1, 6.2, 8.1, 8.5 |
| **LO 3.4B:** Apply definite integrals to problems involving the average value of a function. | 6.3 |
| **LO 3.4C:** Apply definite integrals to problems involving motion. | 6.1, 8.1, 11.1–3 |
| **LO 3.4D:** Apply definite integrals to problems involving area, volume, (BC) and length of a curve. | 8.2, 8.3, 8.4 |
| **LO 3.4E:** Use the definite integral to solve problems in various contexts. | 6.1, 8.1, 8.5 |
| **EU 3.5:** Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change. | |
| **LO 3.5A:** Analyze differential equations to obtain general solutions. | 7.1, 7.4, 7.5 |
| **LO 3.5B:** Interpret, create, and solve differential equations from problems in context. | 7.1, 7.4, 7.5 |

| Big Idea 4: Series (BC) | *Sections* |
|---|---|
| **EU 4.1:** The sum of an infinite number of real numbers may converge. | |
| **LO 4.1A** Determine whether a series converges or diverges. | 9.1, 10.1, 10.4, 10.5 |
| **LO 4.1B:** Determine or estimate the sum of a series. | 10.1 |
| **EU 4.2:** A function can be represented by an associated power series over the interval of convergence for the power series. | |
| **LO 4.2A:** Construct and use Taylor polynomials. | 10.2, 10.3 |
| **LO 4.2B:** Write a power series representing a given function. | 10.1–3 |
| **LO 4.2C:** Determine the radius and interval of convergence of a power series. | 10.4, 10.5 |

For the most current AP* Exam Topic correlation for this textbook, visit PearsonSchool.com/AdvancedCorrelations.

## Using the Book for Maximum Effectiveness

So, how can this book help you to join in the game of mathematics for a winning future? Let us show you some unique tools that we have included in the text to help prepare you not only for the AP Calculus exam, but also for success beyond this course.




**Chapter Openers** provide a photograph and application to show you an example that illustrates the relevance of what you'll be learning in the chapter.

A **Chapter Overview** then follows to give you a sense of what you are going to learn. This overview provides a roadmap of the chapter as well as tells how the different topics in the chapter are connected under one big idea. It is always helpful to remember that mathematics isn't modular, but interconnected, and that the different skills you are learning throughout the course build on one another to help you understand more complex concepts.

**CHAPTER 7 Overview**

One of the early accomplishments of calculus was predicting the future position of a planet from its present position and velocity. Today this is just one of many situations in which we deduce everything we need to know about a function from one of its known values and its rate of change. From this kind of information, we can tell how long a sample of radioactive polonium will last; whether, given current trends, a population will grow or become extinct; and how large major league baseball salaries are likely to be in the year 2020. In this chapter, we examine the analytic, graphical, and numerical techniques on which such predictions are based.

**You will be able to** use slope fields to analyze solution curves to differential equations, and you will be able to use Euler's method to construct solutions numerically.

- Differential equations
- General and particular solutions of differential equations
- Solving exact differential equations
- Slope fields
- Euler's Method

Similarly, the **Objectives and Topics** feature gives you the big ideas in each section and explains their purpose. You should read this as you begin the section and always review it after you have completed the section to make sure you understand all of the key topics that you have just studied.

**Margin Notes** appear throughout the book on various topics. Some notes provide more information on a key concept or an example. Other notes offer practical advice on using your graphing calculator to obtain the most accurate results.

**Differential Equation Mode**

If your calculator has a *differential equation mode* for graphing, it is intended for graphing slope fields. The usual "Y=" turns into a "$dy/dx =$ " screen, and you can enter a function of $x$ and/or $y$. The grapher draws a slope field for the differential equation when you press the GRAPH button.

Brief **Historical Notes** present the stories of people and the research that they have done to advance the study of mathematics. Reading these notes will often provide you with a deeper appreciation for calculus as a human achievement and may inspire you to do great things yourself some day.

*J. Ernest Wilkins, Jr. (1923–2011)*

By the age of nineteen, J. Ernest Wilkins had earned a Ph.D. degree in Mathematics from the University of Chicago. He then taught, served on the Manhattan project (the goal of which was to build the first atomic bomb), and worked as a mathematician and physicist for several corporations. In 1970, Dr. Wilkins joined the faculty at Howard University and served as head of the electrical engineering, physics, chemistry, and mathematics departments before retiring. He was also Distinguished Professor of Applied Mathematics and Mathematical Physics at Clark Atlanta University.

**SOLUTION**

**Model** Using Newton's Law of Cooling with $T_s = 18$ and $T_0 = 98$, we have

$$T - 18 = (98 - 18)e^{-kt} \quad \text{or} \quad T = 18 + 80e^{-kt}.$$

To find $k$ we use the information that $T = 38$ when $t = 5$.

$$38 = 18 + 80e^{-5k}$$
$$e^{-5k} = \frac{1}{4}$$
$$-5k = \ln\frac{1}{4} = -\ln 4$$
$$k = \frac{1}{5}\ln 4$$

The egg's temperature at time $t$ is $T = 18 + 80e^{-(0.2 \ln 4)t}$.

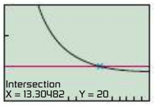
**Solve Graphically** We can now use a grapher to find the time when the egg's temperature is 20°C. Figure 7.12 shows that the graphs of

$$y = 20 \quad \text{and} \quad y = T = 18 + 80e^{-(0.2 \ln 4)t}$$

intersect at about $t = 13.3$.

**Interpret** The egg's temperature will reach 20°C in about 13.3 min after it is put in the pan under running water to cool. Because it took 5 min to reach 38°C, it will take slightly more than 8 additional minutes to reach 20°C.

*Now Try Exercise 31.*




[0, 20] by [10, 40]

**Figure 7.12** The egg will reach 20°C about 13.3 min after being placed in the pan to cool. (Example 6)

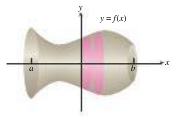
Many examples make use of multiple representations of functions (algebraic, graphical, and numerical) to highlight the different ways of looking at a problem and to highlight the insights that different representations can provide. You should be able to use different approaches for finding solutions to problems. For instance, you would obtain a solution algebraically when that is the most appropriate technique to use, and you would obtain solutions graphically or numerically when algebra is difficult or impossible to use. We urge you to solve problems by one method, then support or confirm your solution by using another method, and finally, interpret the results in the context of the problem. Doing so reinforces the idea that to understand a problem fully, you need to understand it algebraically, graphically, and numerically whenever possible.

Each example ends with a suggestion to **Now Try** a related exercise. Working the suggested exercise is an easy way for you to check your comprehension of the material while reading each section, instead of waiting until the end of each section or chapter to see if you "got it." True comprehension of the textbook is essential for your success on the AP Exam.

**Explorations** appear throughout the text and provide you with the perfect opportunity to become an active learner and discover mathematics on your own. Honing your critical thinking and problem-solving skills will ultimately benefit you on all of your AP Exams.

**EXPLORATION 2** Surface Area

We know how to find the volume of a solid of revolution, but how would we find the *surface area*? As before, we partition the solid into thin slices, but now we wish to form a Riemann sum of approximations to *surface areas of slices* (rather than of volumes of slices).




A typical slice has a surface area that can be approximated by $2\pi \cdot f(x) \cdot \Delta s$, where $\Delta s$ is the tiny *slant height* of the slice. We will see in Section 8.4, when we study arc length, that $\Delta s = \sqrt{\Delta x^2 + \Delta y^2}$, and that this can be written as $\Delta s = \sqrt{1 + (f'(x_k))^2}\,\Delta x$.

Thus, the surface area is approximated by the Riemann sum

$$\sum_{k=1}^{n} 2\pi f(x_k)\sqrt{1 + (f'(x_k))^2}\,\Delta x.$$

1. Write the limit of the Riemann sums as a definite integral from $a$ to $b$. When will the limit exist?
2. Use the formula from part 1 to find the surface area of the solid generated by revolving a single arch of the curve $y = \sin x$ about the $x$-axis.
3. The region enclosed by the graphs of $y^2 = x$ and $x = 4$ is revolved about the $x$-axis to form a solid. Find the surface area of the solid.

Each exercise set begins with a **Quick Review** to help you review skills needed in the exercise set, reminding you again that mathematics is not modular. Each Quick Review includes section references to show where these skills were covered earlier in the text. If you find these problems overly challenging, you should go back through the book and your notes to review the material covered in previous chapters. Remember, you need to *understand* the material from the *entire* calculus course for the AP Calculus Exam, not just memorize the concepts from the last part of the course.

**Quick Review 7.3** *(For help, go to Sections 4.3 and 4.4.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–4, find $dy/dx$.

**1.** $y = x^3 \sin 2x$ **2.** $y = e^{2x} \ln(3x + 1)$

**3.** $y = \tan^{-1} 2x$ **4.** $y = \sin^{-1}(x + 3)$

In Exercises 5 and 6, solve for $x$ in terms of $y$.

**5.** $y = \tan^{-1} 3x$ **6.** $y = \cos^{-1}(x + 1)$

**7.** Find the area under the arch of the curve $y = \sin \pi x$ from $x = 0$ to $x = 1$.

**8.** Solve the differential equation $dy/dx = e^{2x}$.

**9.** Solve the initial value problem $dy/dx = x + \sin x$, $y(0) = 2$.

**10.** Use differentiation to confirm the integration formula

$$\int e^x \sin x\, dx = \frac{1}{2}e^x(\sin x - \cos x).$$

**Standardized Test Questions**

You may use a graphing calculator to solve the following problems.

**47. True or False** If $dy/dx = ky$, then $y = e^{kx} + C$. Justify your answer.

**48. True or False** The general solution to $dy/dt = 2y$ can be written in the form $y = C(3^{kt})$ for some constants $C$ and $k$. Justify your answer.

**49. Multiple Choice** A bank account earning continuously compounded interest doubles in value in 7.0 years. At the same interest rate, how long would it take the value of the account to triple?

(A) 4.4 years (B) 9.8 years (C) 10.5 years
(D) 11.1 years (E) 21.0 years

**50. Multiple Choice** A sample of Ce-143 (an isotope of cerium) loses 99% of its radioactive matter in 199 hours. What is the half-life of Ce-143?

(A) 4 hours (B) 6 hours (C) 30 hours
(D) 100.5 hours (E) 143 hours

For exercises designed to be solved *without a calculator* the numbers of these exercises are printed in a grey oval. We encourage you to *support* the answers to exercises graphically or numerically when you can, but only after you have solved them with pencil and paper.

Along with the standard types of exercises, including skill-based, application, writing, exploration, and extension questions, each exercise set includes a group of **Standardized Test Questions.** Each group includes two true-false with justifications and four multiple-choice questions, with instructions about the permitted use of your graphing calculator.

**CHAPTER 6 Key Terms**

accumulator function (p. 288)
area under a curve (p. 285)
average value (p. 295)
bounded function (p. 289)
cardiac output (p. 275)
characteristic function of the rationals (p. 290)
definite integral (p. 283)
differential calculus (p. 269)
dummy variable (p. 284)
error bounds (p. 319)
Fundamental Theorem of Calculus, Antiderivative Part (p. 302)
Fundamental Theorem of Calculus, Evaluation Part (p. 307)
integrable function (p. 283)
integral calculus (p. 269)
Integral Evaluation Theorem (p. 307)
integral of $f$ from $a$ to $b$ (p. 284)
integral sign (p. 284)
integrand (p. 284)
lower bound (p. 294)
lower limit of integration (p. 284)
LRAM (p. 272)
mean value (p. 295)
Mean Value Theorem for Definite Integrals (p. 296)
MRAM (p. 272)
net area (p. 286)
NINT (p. 289)
norm of a partition (p. 282)
partition (p. 281)
Rectangular Approximation Method (RAM) (p. 272)
regular partition (p. 283)
Riemann sum (p. 281)
Riemann sum for $f$ on the interval $[a, b]$ (p. 282)
RRAM (p. 272)
sigma notation (p. 281)
Simpson's Rule (p. 317)
subinterval (p. 282)
total area (p. 308)
Trapezoidal Rule (p. 315)
upper bound (p. 294)
upper limit of integration (p. 284)
variable of integration (p. 284)

**CHAPTER 6 Review Exercises**

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

The collection of exercises marked in red could be used as a chapter test.

Exercises 1–6 refer to the region $R$ in the first quadrant enclosed by the $x$-axis and the graph of the function $y = 4x - x^3$.

**1.** Sketch $R$ and partition it into four subregions, each with a base

**5.** Sketch the trapezoids and compute (by hand) the area for the $T_4$ approximation.

**6.** Find the exact area of $R$ by using the Fundamental Theorem of Calculus.

**7.** Use a calculator program to compute the RAM approximations in the following table for the area under the graph of $y = 1/x$ from $x = 1$ to $x = 5$.

Each chapter concludes with a list of **Key Terms,** with references back to where they are covered in the chapter, as well as **Chapter Review Exercises** to check your comprehension of the chapter material.

The **Quick Quiz for AP* Preparation** provides another opportunity to review your understanding as you progress through each chapter. A quiz appears after every two or three sections and asks you to answer questions about topics covered in those sections. Each quiz contains three multiple-choice questions and one free-response question of the AP* type. This continual reinforcement of ideas steers you away from rote memorization and toward the conceptual understanding needed for the AP* Calculus Exam.

**Quick Quiz: Sections 7.4 and 7.5**

You may use a graphing calculator to solve the following problems.

**1. Multiple Choice** The rate at which acreage is being consumed by a plot of kudzu is proportional to the number of acres already consumed at time $t$. If there are 2 acres consumed when $t = 1$ and 3 acres consumed when $t = 5$, how many acres will be consumed when $t = 8$?

**(A)** 3.750 **(B)** 4.000 **(C)** 4.066 **(D)** 4.132 **(E)** 4.600

**2. Multiple Choice** Let $F(x)$ be an antiderivative of $\cos(x^2)$. If $F(1) = 0$, then $F(5) =$

**(A)** $-0.099$ **(B)** $-0.153$ **(C)** $-0.293$ **(D)** $-0.992$ **(E)** $-1.833$

**3. Multiple Choice** $\int \frac{dx}{(x-1)(x+3)} =$

**(A)** $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$ **(B)** $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$

**(C)** $\frac{1}{2}\ln|(x-1)(x+3)| + C$ **(D)** $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$

**(E)** $\ln|(x-1)(x+3)| + C$

**4. Free Response** A population is modeled by a function $P$ that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{10}\right).$$

**(a)** If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?

**(b)** If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

**(c)** A different population is modeled by a function $Y$ that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{10}\right).$$

Find $Y(t)$ if $Y(0) = 3$.

**(d)** For the function $Y$ found in part (c), what is $\lim_{t\to\infty} Y(t)$?

**AP* Examination Preparation**

**56.** Consider the infinite region $R$ in the first quadrant under the curve $y = xe^{-x/2}$.

**(a)** Write the area of $R$ as an improper integral.

**(b)** Express the integral in part (a) as a limit of a definite integral.

**(c)** Find the area of $R$.

**57.** The infinite region in the first quadrant bounded by the coordinate axes and the curve $y = \frac{1}{x} - 1$ is revolved about the $y$-axis to generate a solid.

**(a)** Write the volume of the solid as an improper integral.

**(b)** Express the integral in part (a) as a limit of a definite integral.

**(c)** Find the volume of the solid.

**58.** Determine whether or not $\int_0^\infty xe^{-x}\,dx$ converges. If it converges, give its value. Show your reasoning.

An **AP* Examination Preparation** section appears at the end of each set of Chapter Review Exercises and includes three free-response questions of the AP type. This set of questions, which also may or may not permit the use of your graphing calculator, gives you additional opportunity to practice skills and problem-solving techniques needed for the AP Calculus Exam.

In addition to this text, *Pearson Education AP* Test Prep Series: AP* Calculus,* written by experienced AP teachers, is also available to help you prepare for the AP Calculus Exam. What does it include?

- **Text-specific correlations** between key AP test topics and *Calculus: Graphical, Numerical, Algebraic*
- Reinforcement of the important **connections** between what you'll learn and what you'll be tested on in May
- 2 full **sample AB exams** & 2 **sample BC exams**, including answers and explanation
- Test-taking **strategies**

You can order *Pearson Education AP* Test Prep Series: AP* Calculus* by going online to PearsonSchool.com or calling 1-800-848-9500 and requesting ISBN 0-13-331458-8.